Partially coherent vortex beams with a separable phase

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We propose and experimentally implement a method for the generation of a wide class of partially spatially coherent vortex beams whose cross-spectral density has a separable functional form in polar coordinates. We study phase singularities of the spectral degree of coherence of the new beams.   © 2003 Optical Society of America

OCIS codes: 030.0030, 030.4070, 350.5030, 260.3160.

The pioneering work of Nye and Berry on phase singularities in optical fields[1] has revealed the existence of such interesting structures as phase dislocations and optical vortices, which have now become one of the central topics of singular optics.[2] To date, the research on optical vortices has been, for the most part, restricted to the domain of monochromatic beams.[3] Only recently have particular classes of partially coherent beams with wave-front singularities been studied theoretically.[4−6]

In this Letter we present a method for the experimental realization of a wide class of partially spatially coherent singular beams.[7] The cross-spectral density, $W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, \omega)$, of such beams at frequency ω at a pair of points specified by vectors $\boldsymbol{\rho} = (\rho, \phi)$ and $\boldsymbol{\rho}' = (\rho', \phi')$ in any transverse plane $z = \mathrm{const}$ has a separable form in polar coordinates, $W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, \omega) \propto e^{\mathrm{im}(\phi - \phi')} f(\rho, \rho', z, \omega)$. The pairs of points where the function $f(\rho, \rho', z, \omega)$ has zero value specify the positions of phase singularities of the cross-spectral density of such beams. We demonstrate that in any transverse plane the cross-spectral density of these beams has a vortex structure with the axially symmetric absolute value and with a separable phase that has circular edge dislocations. Phase singularities of the correlation functions of optical waves appear to be a relatively new topic in statistical singular optics whose exploration may lead to new interesting developments in this emerging field.

The cross-spectral density of any partially coherent singular beam with a separable phase at a pair of points specified by the transverse vectors $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ in the plane $z = \mathrm{const}$ can be represented as a Mercer-type series in the Laguerre–Gaussian (LG) modes, namely, as a particular case of the coherent-mode expansion[8]

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, \omega) = \sum_{n=0}^{N} \lambda_{nm} \psi_{nm}^{*}(\boldsymbol{\rho}, z, \omega) \psi_{nm}(\boldsymbol{\rho}', z, \omega), \quad (1)$$

where $\lambda_{nm} \geq 0$ are the modal weights. Each LG mode can be expressed as[9]

$$\psi_{nm}(\boldsymbol{\rho}, z, \omega) \propto \left(\frac{w_0}{w_z}\right)\left(\frac{\rho}{w_z}\right)^{|m|} L_n^{|m|}\left(\frac{2\rho^2}{w_z^2}\right)$$
$$\times \exp[-i\Phi(\rho, z)]\exp(-\rho^2/w_z^2). \quad (2)$$

Here $L_n^m(x)$ is the Laguerre polynomial of order n with the azimuthal mode index m; $w_z = (w_0^2 + 4z^2/k^2 w_0^2)^{1/2}$ and $R_z = z + k^2 w_0^4/4z$ are the spot size and the radius of curvature, respectively, of the beam at distance z from its waist; $k = 2\pi/\lambda$ is the wave number, and w_0 is a spot size at the waist of the beam. In Eq. (2), the phase, Φ, can be expressed in the form

$$\Phi(\rho, z) = m\phi - kz + Q \arctan\left(\frac{z}{z_d}\right) - \frac{k\rho^2}{2R_z}, \quad (3)$$

where $Q = 2n + |m| + 1$ and $z_d = k w_0^2/2$ is the characteristic diffraction length of each LG mode.

The separability of the phase of the cross-spectral density of these beams determines the nature of phase singularities of the spectral degree of coherence μ of such beams, defined by the expression[8]

$$\mu(\boldsymbol{\rho}, \boldsymbol{\rho}', z, \omega) \equiv \frac{W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, \omega)}{\sqrt{I(\boldsymbol{\rho}, z, \omega)}\sqrt{I(\boldsymbol{\rho}', z, \omega)}}. \quad (4)$$

Here $I(\boldsymbol{\rho}, z, \omega) \equiv W(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)$ is the spectral intensity of the beam. Given the coordinates of the reference

point, $\boldsymbol{\rho}_0 = (\rho_0, \phi_0)$, in the transverse plane $z = \mathrm{const}$ the positions $\boldsymbol{\rho}_i$ of the phase singularities of μ can be found by solving the equation

$$|\mu(\boldsymbol{\rho}_i, \boldsymbol{\rho}_0, z, \omega)| = 0 . \qquad (5)$$

It follows at once from the axial symmetry of the spectral degree of coherence of the new beams and from Eq. (5) that the only possible phase singularities of such beams are circular edge dislocations[1,2] that occur in any transverse plane $z = \mathrm{const}$ at $\rho = \rho_i(z)$, where the index i labels the edge dislocations. It follows from Eqs. (1)–(4) that the phase of the spectral degree of coherence, $\mu \equiv |\mu| e^{i\Psi}$, of the field at a pair of points with coordinates $(\boldsymbol{\rho}, z)$ and $(\boldsymbol{\rho}_0, z)$ is given by the expression

$$\Psi(\boldsymbol{\rho}, z) = \phi - \phi_0 - \frac{k(\rho^2 - \rho_0^2)}{2R_z} + \pi \sum_i \eta[\rho - \rho_i(z)],$$
$$(6)$$

where $\eta(s)$ is a unit step function.[10] This function specifies π jumps of the phase of the spectral degree of coherence at the positions of circular edge dislocations.

Equation (1) indicates a method for the experimental realization of the novel beams. It follows at once from Eqs. (1) and (2) that, to generate a partially coherent singular beam with a separable phase, one has to assemble a suitable array of statistically independent coaxial sources, each generating a LG mode with the same spot size w_z and azimuthal mode index m. In practice, such LG modes can be generated, for example, by illuminating appropriate computer-synthesized gratings with laser light[11] (see also Ref. 3 for other methods). It follows from Eq. (2) as well as from the normalization of the Laguerre polynomials[12] that the modal weight λ_{nm} is proportional to the power P_n (Ref. 13) of the source producing the nth mode, more specifically, $\lambda_{nm} \propto P_n n!/(n + m)!$.

Let us consider the simplest nontrivial case of a partially coherent singular beam with the topological charge $m = 1$ that is composed of two modes, LG_{01} and LG_{11}, with a given ratio of the mode powers P_1/P_0. In Fig. 1 the behavior of the modulus and of the phase of the spectral degree of coherence of such a beam in a particular transverse plane $z = 1.5 z_d$ is displayed. In agreement with our general theory, one can observe the circular edge dislocation at $\rho_d \approx 1.45 w_z$. The phase of the spectral degree of coherence is seen to undergo a jump by π in passing through such a circular dislocation.

The experimental arrangement for generating and testing partially coherent singular beams with a separable phase is shown in Fig. 2. A He–Ne laser generated a Gaussian beam (LG_{00} mode with $\lambda = 0.63$ μm, $w_0 = 1.6$ mm) with longitudinal coherence length l_coh estimated to be equal to 270 mm. On splitting the laser beam into two beams and imposing a delay $\Delta L = 800$ mm $\approx 3 l_\mathrm{coh}$ between the beams, we obtained two uncorrelated LG_{00} beams suitable for generating an incoherent superposition of coaxial LG_{01} and LG_{11} modes with the same spot size w_z.

Following the procedure of Ref. 11, we produced such modes using diffraction of probing beams at the appropriate computer-synthesized holograms, which were placed at the same distance from the beam splitter of a misaligned interferometer. The ratio of the powers of the two modes, $P_1/P_0 = 0.45$, was controlled with neutral attenuators (not shown in Fig. 2) at the arms of the interferometer. Spatial filters behind the gratings selected the modes LG_{01} and LG_{11} with the same sign of the topological charge $m = +1$.

To exhibit a vortex nature of partially coherent singular beams with a separable phase, we performed a Young-type interference experiment in its original version[14] in which a light wave was incident upon an opaque strip and a superposition of the secondary waves diffracted at the edges of the strip, the so-called boundary diffraction waves, produced interference fringes behind the strip. In our experiment we placed the strip at right angle to the direction of propagation of a partially coherent singular beam so that the center of symmetry of the strip lay on the axis of
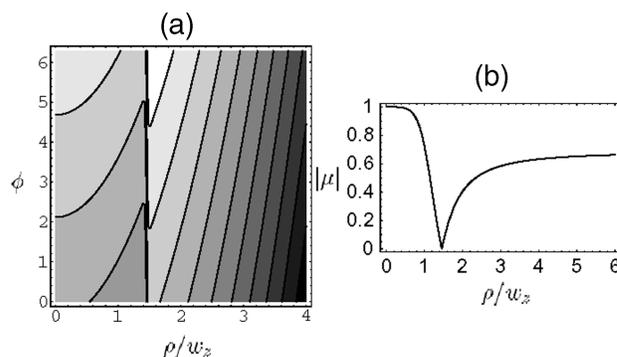

Fig. 1. Properties of partially coherent singular beams composed of LG_{01} and LG_{11} modes: (a) contours of constant phase of the spectral degree of coherence at a pair of points with coordinates (ρ, ϕ, z) and $(0.1 w_z, 0, z)$ in the plane $z = 1.5 z_d$, (b) modulus of the spectral degree of coherence $|\mu|$ at the same pair of points as a function of the dimensionless radial variable ρ/w_z. The ratio of the powers of the two modes is taken to be the same as in the experiment, $P_1/P_0 = 0.45$.
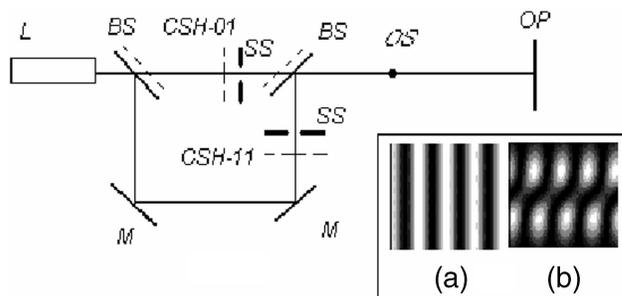

Fig. 2. Experimental arrangement for generating and testing partially coherent singular beams with a separable phase: L, laser; BSs, beam splitters, Ms, mirrors; CSH-01 and CSH-11, computer-synthesized holograms producing LG_{01} and LG_{11} modes, respectively; SSs, spatial selectors of diffraction orders; OS, opaque screen; OP, observation plane. Inset: Young interference fringes behind the opaque strip illuminated by (a) vortex-free LG_{00} mode and (b) LG_{01} mode.
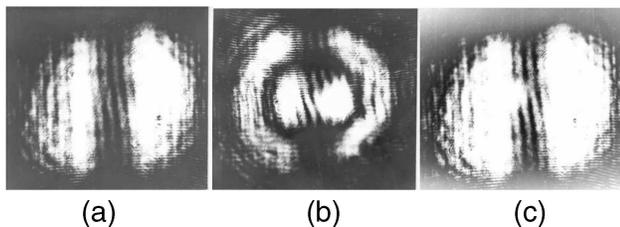
Fig. 3. Experimental results: interference pattern behind the opaque strip illuminated by (a) LG_{01} mode, (b) LG_{11} mode, and (c) incoherent superposition of the LG_{01} and LG_{11} modes. The distance between the strip and the observation plane is 470 mm; the width of strip d is 1 mm, and the width of the beam in the plane of the strip, $w_z \approx 1.62$ mm.

the beam. Such an arrangement made it possible to study coherence properties of beams of this kind at pairs of points with the same radial coordinates $\Delta\rho \equiv \rho - \rho' = 0$ and with the polar angle difference $\Delta\phi \equiv \phi - \phi'$ in the interval $\Delta\phi_{\min} \leq \Delta\phi \leq \pi$, where the value of $\Delta\phi_{\min}$ was determined by the width of the strip. According to the theory of the boundary diffraction wave,[15] we can view the fringes behind the strip as though they were produced by only those wavelets that are associated with the points located at the same height on the opposite edges of the strip. Because of the symmetry of the strip and of the incident beam, the visibility of the interference fringes was equal to unity, within the accuracy of the measurement.

As seen from the inset of Fig. 2, the interference fringes produced by the vortex-free LG_{00} mode, which was normally incident on the strip, are straight lines. However, the interference fringes produced by the $LG_{0\pm 1}$ mode, which carries an optical vortex, are bent. A shift of the fringes by the factor of π, which can clearly be seen in the inset, is associated with the helicity of the $LG_{0\pm 1}$ mode.

In Fig. 3, the interference fringes behind the strip illuminated by modes LG_{01} and LG_{11} alone, as well as by the combined partially coherent beam, are displayed. Bending of the fringes clearly shows that the cross-spectral density of the combined beam has a helical phase with the same topological charge, $m = \pm 1$, as that of the constituent modes. The fringe visibility V was found to be equal to 0.97 ± 0.03 for all pairs of points with $\Delta\rho = 0$, irrespective of the value of $\Delta\phi$. It follows that the modulus of the spectral degree of coherence depends only on the radial variables.

Further, the analysis of the intensity of the interference pattern produced by a partially coherent singular beam composed of the LG_{01} and LG_{11} modes reveals its $\sin^2(\Delta\phi/2)$ dependence on the phase difference $\Delta\phi$. It follows from the expression for the spectral intensity in the observation plane in the Young interference experiment[8] that a periodic dependence of the intensity on $\Delta\phi$, together with the axial symmetry of $|\mu|$, implies that $\mu(\Delta\rho \equiv 0, \phi, \phi') \propto e^{im(\phi - \phi')}$. This dependence is a

signature of a partially spatially coherent optical vortex. Finally, the invariance of the main qualitative features of Fig. 3 with respect to rotation of the strip in the transverse plane of the incident beam has been verified experimentally.

In summary, we have studied theoretically a new class of partially coherent beams whose cross-spectral density at a pair of points in any transverse plane is separable in polar coordinates. We have experimentally demonstrated the vortex structure of the cross-spectral density of such beams. We have found that the only possible phase singularities of the spectral degree of coherence of the new beams are circular edge dislocations that occur in any transverse plane. The invariance of circular edge dislocations of these beams on propagation in free space might find applications in optical computing and in connection with the transfer of optical information through inhomogeneous media.

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